Local Inertial Coordinate System and The Principle of Equivalence

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Abstract

In this paper, the local inertial coordinate system is calculated through coordinate transformations from laboratory coordinate system. We derived the same free falling equations as those in General Relativity. However, the definitions of second and higher order covariant derivatives are different. Our results are different from the classic view of the principle of equivalence, and suggest that all the laws of physics in gravitational field can be given by doing coordinate transformations from local inertial coordinate system to lab coordinate system.

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I. INTRODUCTION

Einstein's General Relativity are in good agreement with the solar observation data, but it also meets great challenges in galactic and larger scales. So either we have to accept the existence of dark matter and dark energy, or we must go another way to modify the theory of Gravitation[1][2][3][4]. We here, motivated by a theoretical approach, reconsidered the translation of equivalence principle, the foundation of General Relativity. It is well-known that General Relativity is based on the principle of equivalence, "*at every space-time point in an arbitrary gravitational field it is possible to choose a 'locally inertial coordinate system' such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation*"[5]. Equivalence principle describes the gravitational effects as coordinate transformations, but General Relativity does not. Our ultimate aim is to establish a coordinate theory of gravitation. As a modest step, we here give the LICS (local inertial coordinate system) a full description, and the further topic concerning the gravitational field equations will be given in a forth coming paper[6]. In our opinion, all the gravitational effects can be described by the LICS, and the gravitational equations should also have great relations to the LICS, because the gravitational equations should become trivial if we transform the coordinates from lab coordinate system to LICS. So it is of essential importance to find the coordinate transformations between the LICS and the lab coordinate system.

In the theory of General Relativity, the LICS at a spacetime point $\mathcal{P}_0$ is treated as the "closest thing"[7] to global Lorentz frame:

$$\begin{cases} g_{\mu\nu}(\mathcal{P}_0) = \eta_{\mu\nu} \\ g_{\mu\nu,\alpha}(\mathcal{P}_0) = 0 \end{cases} \tag{1}$$

$g_{\mu\nu,\alpha,\beta}(\mathcal{P}_0)$ is non-zero due to the spacetime curvature. In this paper we should like to point out that we find the LICS in one point of gravitation is the "same thing" to global Lorentz frame. Symbols denoting local coordinate transformations instead of metric tensors are used during calculations. All the laws of physics in gravitational field can be given by doing a certain coordinate transformation, so the gravitational effects are treated totally as some kind of coordinate transformations. It will be demonstrated that the free falling equations

of a testing neutral particle are in coincidence with the well known ones

$$\frac{dU^\lambda}{d\tau} + \left\{ \begin{matrix} \lambda \\ \mu \ \nu \end{matrix} \right\} U^{\mu\nu} = 0, \tag{2}$$

in which $\left\{ \begin{matrix} \lambda \\ \mu \ \nu \end{matrix} \right\}$ is the Christoffel symbol. And we will continue show a different structure of the second and higher order covariant derivatives through coordinate transformations. The form of the second order covariant derivatives is given and higher orders can be calculated in a straightforward way. We also find that the Riemann curvature tensor $R_{\kappa\omega\mu\nu}$, as the description of spacetime curvature, is not sufficient. It is only a rank 2 description, and a full definition is given in this paper. Given a point particle p of infinitesimal mass in gravitational field, in the LICS of p, the gravitational field equation should be second order differential equation. With a coordinate transformation to lab coordinate system, we can find the contribution of particle p to the gravitational field. We do not concentrate on this point here, which is rather a complicated problem, and it will be further discussed in [6]. In general, we would like to emphasize the importance of finding a detailed description of LICS and hence a different structure of covariant derivatives.

II.   THE FREE FALLING EQUATION

In developing our new interpretation we start with a coordinate transformation

$$\theta^\alpha_{\ \mu}(x) \equiv \frac{\partial \xi^\alpha}{\partial x^\mu}(x) = \Lambda^\alpha_{\ \beta}(x) X^\beta_{\ \mu}(x). \tag{3}$$

Here $\{\xi^\alpha\}$ are the local Lorentz coordinates and $\{x^\mu\}$ are the coordinates used by distant observers. $\Lambda^\alpha_{\ \beta}(x)$ are the local Lorentz transformations, and $X^\beta_{\ \mu}(x)$ denotes the remain part of the coordinate transformations. Similarly we can also define a reverse transformation of the coordinates

$$e^\mu_{\ \alpha}(x) \equiv \theta^{-1\mu}_{\quad \alpha}(x) = \frac{\partial x^\mu}{\partial \xi^\alpha} = X^{-1\mu}_{\quad \beta}(x)\Lambda_\alpha^{\ \beta} \tag{4}$$

in which $X^{-1\mu}_{\quad \beta}$ is the inverse matrix of $X^\beta_{\ \mu}$, so is the matrix $\Lambda^{-1\beta}_{\quad \alpha} = \Lambda_\alpha^{\ \beta} = \eta_{\alpha\gamma}\Lambda^\gamma_{\ \delta}\eta^{\delta\beta}$. The X (or X^{-1}) part of coordinate transformations makes inertial coordinates into non-inertial ones, so gravitational redshifts are embed in the first introduced terms $X^\beta_{\ \mu}$ and

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$X^{-1\mu}{}_{\beta}$, which also include arbitrary coordinate transformations of the observers. In the gravitational field, the LICS can only exist within a sufficiently small region of any point, as a result, a free falling inertial coordinate system in the gravitational field must have been "twisted". So the $X^{\beta}{}_{\mu}$ should include not only the gravitational redshifts terms $Z^{\lambda}{}_{\mu}$ but also inertial coordinate twist terms (ICT) $\tilde{Z}^{\alpha}{}_{\lambda}$. Here the first introduced ICT terms are different from the stretch or crush effects of redshifts terms, and can be understood as wrinkle effects on the free falling coordinates due to the inhomogeneous property of gravitational field. And they can only make sense in gravitational field and should be trivial($\tilde{Z}^{\beta}{}_{\lambda} = \delta^{\beta}{}_{\lambda}$) in flat spacetime. Generally speaking, $X^{\beta}{}_{\mu}$ can be written as $\tilde{Z}^{\beta}{}_{\lambda}Z^{\lambda}{}_{\mu}$.

Because $\theta^{\alpha}{}_{\mu}(x)$ denotes coordinate transformations between LICS and the laboratory frame, we here give a requirement:

$$\begin{cases} \theta^{\alpha}{}_{\mu,\nu} &= \theta^{\alpha}{}_{\nu,\mu} \\ e^{\mu}{}_{\alpha,\beta} &= e^{\mu}{}_{\beta,\alpha} \end{cases} \tag{5}$$

It is quite natural to require that the coordinate transformations are smooth and continuous, which can also be considered as a basic assumption about nature. Now using eq. (5) we can derive the relationship between $\Lambda^{\alpha}{}_{\beta,\lambda}(x)$ and $X^{\alpha}{}_{\mu,\lambda}(x)$:

$$\Lambda^{\alpha}{}_{\beta,\nu} = \frac{1}{2}\Lambda^{\alpha}{}_{\gamma}\eta^{\gamma\delta}(X^{-1\mu}{}_{\beta}T_{\delta\nu\mu} + X^{-1\lambda}{}_{\delta}T_{\beta\lambda\nu} - X^{-1\mu}{}_{\beta}X^{-1\lambda}{}_{\delta}X^{\sigma}{}_{\nu}T_{\sigma\mu\lambda}) \tag{6}$$

in which $T_{\alpha,\nu,\mu} \equiv X_{\alpha\nu,\mu} - X_{\alpha\mu,\nu}$. To look into the interesting physics of the relatively complicated relation (6) more clearly, we can choose some special coordinate conditions, i.e., $X^{\alpha}{}_{\mu} = \delta^{\alpha}{}_{\mu}$, eq. (6) becomes:

$$\Lambda^{\alpha}{}_{\beta,\nu} = \Lambda^{\alpha}{}_{\gamma}\eta^{\gamma\delta}(X_{\delta\nu,\beta} - X_{\beta\nu,\delta}) \tag{7}$$

Continue requiring $\Lambda^{\alpha}{}_{\gamma} = \delta^{\alpha}{}_{\gamma}$, we finally get:

$$\Lambda_{\alpha\beta,\nu} = X_{\alpha\nu,\beta} - X_{\beta\nu,\alpha} \tag{8}$$

We can see that $\Lambda^{\alpha}{}_{\beta,\nu}$ is totally determined by $X^{\alpha}{}_{\mu,\nu}$. And if we choose $\alpha = 0, \beta = 1, \nu = 0$, in the static Gravitational field, the l.h.s. of eq. (8) is just acceleration, while the r.h.s. is the derivative of gravitational potential with respect to x^1. That is to say, the acceleration of a free falling particle in static gravitational field at the instant of stationary is determined by the gratitude of the gravitational potential.

The compatibility condition eq. (6) gives no constraint on $X_{\mu\nu}$. In order to get back to physical results in flat spacetime, so here, before other compatibility conditions are introduced, $\tilde{Z}_\lambda^\beta$ can be identified as δ_λ^β and $\tilde{Z}_{\lambda,\rho}^\beta$ can be set to zero. That is to say, $X_{\mu\nu}$ $(X_{\mu\nu,\lambda})|_{\tilde{Z}=\delta}$ equals to $Z_{\mu\nu}$ $(Z_{\mu\nu,\lambda})$. Having got the relationship (6) between the derivatives of $\Lambda^\alpha_{\ \beta}$'s and $Z^\alpha_{\ \mu}$'s, we can prove that the affine connection $\Gamma^\lambda_{\mu\nu} = e^\lambda_{\ \alpha}\theta^\alpha_{\ \mu,\nu}$ takes the value of Christoffel Symbol in the free falling equation

$$\frac{dU^\lambda}{d\tau} + e^\lambda_{\ \alpha}\theta^\alpha_{\ \mu,\nu}U^{\mu\nu} = 0 \tag{9}$$

The proof is quite easy. Substituting the derivatives of $\Lambda^\alpha_{\ \beta}$ derived in eq. (6) into the definition of $\Gamma^\lambda_{\mu\nu}$, then using the relation $g_{\mu\nu} \equiv Z^\alpha_{\ \mu}\eta_{\alpha\beta}Z^\beta_{\ \nu}$, we finally get the results:

$$\Gamma^\lambda_{\mu\nu}|_{\tilde{Z}=\delta}$$
$$= e^\lambda_{\ \alpha}\theta^\alpha_{\ \mu,\nu}|_{\tilde{Z}=\delta}$$
$$= Z^{-1\lambda}_{\ \ \gamma}\Lambda_\alpha^{\ \gamma}(\Lambda^\alpha_{\ \beta}Z^\beta_{\ \mu})_{,\nu}$$
$$= Z^{-1\lambda}_{\ \ \beta}Z^\beta_{\ \mu,\nu} + Z^{-1\lambda}_{\ \ \gamma}\Lambda_\alpha^{\ \gamma}\Lambda^\alpha_{\ \beta,\nu}Z^\beta_{\ \mu}$$
$$= Z^{-1\lambda}_{\ \ \beta}Z^\beta_{\ \mu,\nu} + \frac{1}{2}Z^{-1\lambda}_{\ \ \gamma}\eta^{\delta\gamma}(Z^{-1\omega}_{\ \ \beta}T_{\delta\nu\omega} + Z^{-1\xi}_{\ \ \delta}T_{\beta\xi\nu} - Z^{-1\omega}_{\ \ \beta}Z^{-1\xi}_{\ \ \delta}Z^\sigma_{\ \nu}T_{\sigma\omega\xi})Z^\beta_{\ \mu}$$
$$= Z^{-1\lambda}_{\ \ \beta}Z^\beta_{\ \mu,\nu} + \frac{1}{2}Z^{-1\lambda\delta}Z_{\delta\nu,\mu} - \frac{1}{2}Z^{-1\lambda\delta}Z_{\delta\mu,\nu} + \frac{1}{2}Z^{-1\lambda\delta}Z^{-1\xi}_{\ \ \delta}Z^\beta_{\ \mu}Z_{\beta\xi,\nu} - \frac{1}{2}Z^{-1\lambda\delta}Z^\beta_{\ \mu}Z_{\beta\nu,\delta}$$
$$\quad - \frac{1}{2}Z^{-1\lambda\delta}Z^\sigma_{\ \nu}Z_{\sigma\mu,\delta} + \frac{1}{2}Z^{-1\lambda\delta}Z^{-1\xi}_{\ \ \delta}Z^\beta_{\ \nu}Z_{\beta\xi,\mu}$$
$$= \frac{1}{2}g^{\lambda\xi}[(Z^\alpha_{\ \xi}\eta_{\alpha\beta}Z^\beta_{\ \mu,\nu} + Z^\beta_{\ \mu}\eta_{\alpha\beta}Z^\alpha_{\ \xi,\nu}) + (Z^\alpha_{\ \xi}\eta_{\alpha\beta}Z^\beta_{\ \nu,\mu} + Z^\alpha_{\ \xi,\mu}\eta_{\alpha\beta}Z^\beta_{\ \nu}) - (Z^\alpha_{\ \mu}\eta_{\alpha\beta}Z^\beta_{\ \nu,\xi} + Z^\beta_{\ \nu}\eta_{\alpha\beta}Z^\alpha_{\ \mu,\xi})]$$
$$= \frac{1}{2}g^{\lambda\xi}(g_{\xi\mu,\nu} + g_{\xi\nu\mu} - g_{\mu\nu,\xi})$$
$$= \left\{ \begin{array}{c} \lambda \\ \mu\ \nu \end{array} \right\}$$

In the above proof we used eq. (6) and the definition $g_{\mu\nu} \equiv Z^\alpha_{\ \mu}\eta_{\alpha\beta}Z^\beta_{\ \nu}$. After the above relative lengthly discussion, we derived the same free falling equations as those in General Relativity.

III. DIFFERENTIAL EQUATIONS

In this section we will demonstrate, according to the compatibility conditions of coordinates, $\tilde{Z}_{\beta\lambda}$ and any order of its derivatives can be calculated exactly, provided that $Z_{\lambda\mu}$ and

its any order of derivatives are known. Differentiating with respect to μ on both sides of eq. (6) and requiring $\Lambda^\alpha_{\beta,\mu,\nu} = \Lambda^\alpha_{\beta,\nu,\mu}$, we get the following equation:

$$
\begin{aligned}
&[Z^{-1\rho}{}_\beta(Z^\gamma{}_\mu \tilde{Z}_{\alpha\gamma,\rho,\nu} - Z^\gamma{}_\nu \tilde{Z}_{\alpha\gamma,\rho,\mu}) + Z^{-1\rho}{}_\beta(Z^\sigma{}_\mu \tilde{Z}_{\sigma\alpha,\rho,\nu} - Z^\sigma{}_\nu \tilde{Z}_{\sigma\alpha,\rho,\mu}) \\
&+\; Z^{-1\lambda}{}_\alpha(Z^\gamma{}_\nu \tilde{Z}_{\beta\gamma,\lambda,\mu} - Z^\gamma{}_\mu \tilde{Z}_{\beta\gamma,\lambda,\nu}) + Z^{-1\lambda}{}_\alpha(Z^\sigma{}_\nu \tilde{Z}_{\sigma\beta,\lambda,\mu} - Z^\sigma{}_\mu \tilde{Z}_{\sigma\beta,\lambda,\nu})] - W_{\alpha\beta\mu\nu} = 0. \quad (10)
\end{aligned}
$$

In the above equation we introduced a tensor variable $W_{\alpha\beta\mu\nu}$ for convenience. And it is defined as:

$$
\begin{aligned}
W_{\alpha\beta\mu\nu} =\; &[Z^{-1\rho}{}_\beta(Z_{\alpha\nu,\rho,\mu} - Z_{\alpha\mu,\rho,\nu}) + Z^{-1\lambda}{}_\alpha(Z_{\beta\mu,\lambda,\nu} - Z_{\beta\nu,\lambda,\mu}) \\
&+\; Z^{-1\rho}{}_\beta Z^{-1\lambda}{}_\alpha Z^\sigma{}_\mu(Z_{\sigma\rho,\lambda,\nu} - Z_{\sigma\lambda,\rho,\nu}) + Z^{-1\rho}{}_\beta Z^{-1\lambda}{}_\alpha Z^\sigma{}_\nu(Z_{\sigma\lambda,\rho,\mu} - Z_{\sigma\rho,\lambda,\mu})] \quad (11) \\
&+\; [2\Lambda_{\sigma\alpha}\Lambda^{\sigma\delta}{}_{,\mu}\Lambda_{\eta\delta}\Lambda^\eta{}_{\beta,\nu} + Z^{-1\rho}{}_{\beta,\mu}T_{\alpha\nu\rho} + Z^{-1\lambda}{}_{\alpha,\mu}T_{\beta\lambda\nu} - (Z^{-1\rho}{}_\beta Z^{-1\lambda}{}_\alpha Z^\sigma{}_\nu)_{,\mu}T_{\sigma\rho\lambda} - (\mu \leftrightarrow \nu)]
\end{aligned}
$$

It is clear that eq.(10) is a linear equation of $\tilde{Z}_{\mu\nu,\lambda,\rho}$. So we can rewrite the above equation as the form:

$$
\tilde{Z}_{\mu\nu,\lambda,\rho} = L_2(\tilde{Z}_{\mu\nu,\lambda,\rho}), \tag{12}
$$

in which the function L_2 is some kind of linear function. Similar to what we did to eq. (6) and remembering $\tilde{Z}_{\mu\nu,\lambda,\rho,\delta} = \tilde{Z}_{\mu\nu,\lambda,\delta,\rho}$, we get the 3rd order compatibility condition

$$
\tilde{Z}_{\mu\nu,\lambda,\rho,\delta} = L_3(\tilde{Z}_{\mu\nu,\lambda,\rho,\delta}) \tag{13}
$$

Using this iterative method we can get the compatibility conditions of $\tilde{Z}_{\mu\nu}$'s any order derivatives.

$$
\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n} = L_n(\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n}). \tag{14}
$$

We may decompose eq.(14) as follows :

$$
\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n} = \bar{L}_n(\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n}) + Y_n(Z_{\mu\nu}, \cdots, Z_{\mu\nu,\lambda_1...\lambda_n}), \tag{15}
$$

in which $\bar{L}_n$ is the homogeneous term and Y_n is the nonhomogeneous term. When Y_n takes all possible values, the solutions of eq.(15) $\{\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n}\}$ form a group G under the action of tensor addition. While the solutions of the corresponding homogeneous linear equation

$$
\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n} = \bar{L}_n(\tilde{Z}_{\mu\nu,\lambda_1...\lambda_n}) \tag{16}
$$

form a subgroup of G.

Before solving the above equations, we first discuss our definition of spacetime curvature. As we all know, in General Relativity, the curvature of spacetime is described by $R_{\kappa\omega\mu\nu}$. But in our calculations, the definition is not sufficient. The term $\tilde{Z}_{\mu\nu}$ appears when there is spacetime curvature and vanishes in flat spacetime. According to eq.(15), a nonzero $\{Y_n\}$ requires nontrivial $\tilde{Z}_{\mu\nu}$. If $\{Y_n\}$ is zero, Minkowski coordinates is included in this case, $\tilde{Z}_{\mu\nu}$ must equal to $\eta_{\mu\nu}$ to recover the Minkowski spacetime. Hence, at one spacetime point $R_{\kappa\omega\mu\nu} = 0$ is not an accurate definition of flatness, and it should be defined as $\{Y_n\} = 0$. Meanwhile at rank 2 Y_2 is just $W_{\alpha\beta\mu\nu}$, which is equivalent to curvature tensor $R_{\kappa\omega\mu\nu}$(see eq.(17)).

To find the physical solution of the above linear tensor equation eq.(15), we first consider the solution of eq.(16) in which the absence of nonhomogeneous term means that the space time is flat. For the ICT term only appears when there is gravity and it is trivial in flat spacetime, it means that only one trivial solution of eq.(16), $\tilde{Z}_{\mu\nu} = \eta_{\mu\nu}$, is physical. The properties of a group and its subgroup ensure that the physical solution of eq.(15) is unique.

We here in this paper give the solution of L_2 eq.(12). First we derived some properties of the newly defined tensor variable $W_{\alpha\beta\mu\nu}$:

$$
\begin{aligned}
W_{\alpha\beta\mu\nu} &= -W_{\beta\alpha\mu\nu} \\
W_{\alpha\beta\mu\nu} &= -W_{\alpha\beta\nu\mu} \\
W_{\alpha\beta\mu\nu} &= Z^{\gamma}{}_{\mu} Z^{\sigma}{}_{\nu} Z^{-1\rho}{}_{\alpha} Z^{-1\lambda}{}_{\beta} W_{\gamma\sigma\rho\lambda} \\
W_{\alpha\beta\mu\nu} &= \frac{1}{2}\{[Z^{\gamma}{}_{\mu} Z^{-1\rho}{}_{\alpha} W_{\gamma\beta\rho\nu} - (\alpha \leftrightarrow \beta)] - (\mu \leftrightarrow \nu)\} \\
W_{\alpha\beta\mu\nu} &= -2 Z^{-1\kappa}{}_{\alpha} Z^{-1\omega}{}_{\beta} R_{\kappa\omega\mu\nu}
\end{aligned}
\tag{17}
$$

Here the term $R_{\kappa\omega\mu\nu}$ is the Riemann curvature. Using the above relation of $W_{\alpha\beta\mu\nu}$, it is easy to check that the solution of eq.(10) have the form:

$$
\tilde{Z}_{\alpha\mu,\beta,\nu} = \frac{1}{12}(Z^{\rho}{}_{\beta} Z^{-1\lambda}{}_{\mu} W_{\alpha\rho\lambda\nu} + Z^{\rho}{}_{\nu} Z^{-1\lambda}{}_{\mu} W_{\alpha\rho\lambda\beta})
\tag{18}
$$

As all the variables describing coordinate transformations is clear, we can build the coordinate transformation rules in general. The definition of covariant derivative on cotracovariant

tensor is:

$$
\begin{aligned}
A_{\alpha_1\alpha_2\cdots\alpha_n,\beta} \\
= (\frac{\partial x^{\mu_1}}{\partial \xi^{\alpha_1}}\cdots\frac{\partial x^{\mu_n}}{\partial \xi^{\alpha_n}}A_{\mu_1\cdots\mu_n})_{,\nu}\frac{\partial x^\nu}{\partial \xi^\beta} \\
= e^\nu{}_\beta\prod_j e^{\mu_j}{}_{\alpha_j}(A_{\mu_1\cdots\mu_n,\nu} - \sum_{i=1}^n \Gamma^{\rho_i}_{\mu_i\nu}A_{\mu_1\cdots\rho_i\cdots\mu_n}) \\
= e^\nu{}_\beta\prod_j e^{\mu_j}{}_{\alpha_j}A_{\mu_1\cdots\mu_n;\nu}
\end{aligned}
\tag{19}
$$

in which $\Gamma^{\rho_i}_{\mu_i\nu} = e^{\rho_i}{}_\sigma\theta^\sigma{}_{\mu_i,\nu}$ is defined in the free falling equation eq.(9).

For the covariant tensor, a similar calculation can be performed and we come to a similar result:

$$
\begin{aligned}
A^{\alpha_1\alpha_2\cdots\alpha_n}{}_{,\beta} \\
= (\frac{\partial \xi^{\alpha_1}}{\partial x^{\mu_1}}\cdots\frac{\partial \xi^{\alpha_n}}{\partial x^{\mu_n}}A^{\mu_1\cdots\mu_n})_{,\nu}\frac{\partial x^\nu}{\partial \xi^\beta} \\
= e^\nu{}_\beta\prod_j \theta^{\alpha_j}{}_{\mu_j}(A^{\mu_1\cdots\mu_n}{}_{,\nu} + \sum_{i=1}^n \Gamma^{\mu_i}_{\rho_i\nu}A^{\mu_1\cdots\rho_i\cdots\mu_n}) \\
= e^\nu{}_\beta\prod_j \theta^{\alpha_j}{}_{\mu_j}A^{\mu_1\cdots\mu_n}{}_{;\nu}
\end{aligned}
\tag{20}
$$

As we know in General Relativity, the second order covariant differentiation of a vector or tensor field is not symmetric about the differential indices, instead there is an extra term which couples to spacetime curvature,

$$
A_{\mu;\nu;\lambda} = A_{\mu;\lambda;\nu} - A_\sigma R^\sigma{}_{\mu\nu\lambda}.
\tag{21}
$$

But in our case, the term $A_{\mu;\nu;\lambda}$ is just the coordinate transformed form of the second order partial derivative term in local Lorentz coordinate $A_{\alpha,\beta,\gamma}$, and it is certainly symmetric about the factor ν and λ.

$$
\begin{aligned}
A_{\mu;\nu;\lambda} &= \frac{\partial \xi^\alpha}{\partial x^\mu}\frac{\partial \xi^\beta}{\partial x^\nu}\frac{\partial \xi^\gamma}{\partial x^\lambda}A_{\alpha,\beta,\gamma} \tag{22}\\
&= A_{\mu,\nu,\lambda} - \Gamma^\xi_{\mu\nu,\lambda}A_\xi - \Gamma^\xi_{\mu\nu}A_{\xi,\lambda} - \Gamma^\xi_{\mu\lambda}A_{\xi,\nu} + \Gamma^\eta_{\mu\lambda}\Gamma^\xi_{\eta\nu}A_\xi - \Gamma^\xi_{\nu\lambda}(A_{\mu;\xi}) \tag{23}
\end{aligned}
$$

In the above equation we get a very close form as that in General Relativity. The only difference lies in the term $\Gamma^\xi_{\mu\nu,\lambda}$. In our cases, it must be treated very carefully due to the

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contribution of the second order differentiation of $\tilde{Z}$.

$$
\begin{aligned}
\theta^{\alpha}_{\ \mu,\nu,\lambda} &= (\Lambda^{\alpha}_{\ \beta}X^{\beta}_{\ \mu}),_{\nu,\lambda} \\
&= \Lambda^{\alpha}_{\ \beta,\nu,\lambda}X^{\beta}_{\ \mu} + \Lambda^{\alpha}_{\ \beta}(\tilde{Z}^{\beta}_{\ \rho}Z^{\rho}_{\ \mu}),_{\nu,\lambda} + \Lambda^{\alpha}_{\ \beta,\nu}X^{\beta}_{\ \mu,\lambda} + +\Lambda^{\alpha}_{\ \beta,\lambda}X^{\beta}_{\ \mu,\nu}
\end{aligned}
\tag{24}
$$

IV. CONCLUSION

As a general concept, this paper presents a description of LICS, which can be understood as a new interpretation of Equivalence Principle. Having got the right description of LICS, all the calculations concerning gravitational effects are based on coordinate transformations which are required smooth and continuous. The form of $\Gamma^{\lambda}_{\mu\nu}$ can be exactly calculated once the redshifts of spacetime $Z_{\mu\nu}$ is known, but it is not Christoffel symbol as that in General Relativity. Then we derived the same free falling equations as those in General Relativity. However, the definitions of second and higher order covariant derivatives are different from those in General Relativity. Under this interpretation, many concepts of gravitation may be changed including the gravitational field equations. We shall focus on this point in our forthcoming coming paper[6]. In conclusion, our main point in this paper is to give a description of LICS, and all the gravitational effects can be calculated through this coordinate transformation.

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